

Answers to Problem Set 9

Total: 54 marks

Note – in translations, there are many possible answers! *Any answer that's FO equivalent to mine is also fully correct.*

11.16 [1 mark each, total 10 marks]

1. $\forall x (Cube(x) \rightarrow \forall y (Tet(y) \rightarrow LeftOf(x, y)))$

2. $\forall x ((Cube(x) \wedge Small(x)) \rightarrow \exists y (Large(y) \wedge Cube(y) \wedge BackOf(x, y)))$

3. $\exists x (Cube(x) \wedge \forall y (Tet(y) \rightarrow FrontOf(x, y)))$

4. $\exists x (Cube(x) \wedge Large(x) \wedge \exists y (Small(y) \wedge Cube(y) \wedge FrontOf(x, y)))$

5. $\forall x \neg \forall y (Larger(x, y))$

6. $\forall x ((Cube(x) \wedge \forall y (Tet(y) \rightarrow FrontOf(x, y))) \rightarrow Large(x))$

7. $\forall x (\exists y (Large(y) \wedge Cube(y) \wedge RightOf(x, y)) \rightarrow Small(x))$

8. $\forall x ((\exists y (Cube(y) \wedge BackOf(x, y)) \wedge \exists z (Cube(z) \wedge FrontOf(x, z))) \rightarrow \neg Large(x))$

9. $\forall x (\neg \exists y BackOf(y, x) \rightarrow Cube(x))$

10. $\forall x (Dodec(x) \rightarrow \exists y (Tet(y) \wedge Smaller(x, y)))$

11.18 [2 marks each, total 10 marks]

1. $\forall x (\neg \exists y FrontOf(y, x) \rightarrow Large(x))$

2. $\forall x ((Cube(x) \wedge \exists y FrontOf(y, x)) \rightarrow Small(x))$

3. $\forall x \forall y ((Cube(x) \wedge Dodec(y) \wedge BackOf(x, y)) \rightarrow Smaller(x, y))$

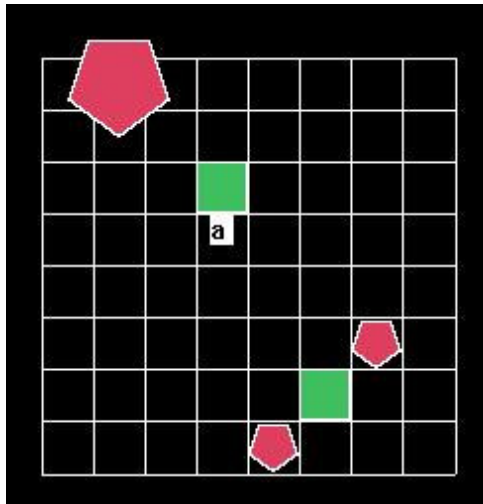
4. $\forall x \forall y (Between(e, x, y) \rightarrow (Small(x) \wedge Small(y)))$

5. $\forall x \forall y (\exists z (Tet(z) \wedge Between(z, x, y)) \rightarrow (Small(x) \wedge Small(y)))$

11.26 (N.B. parts 9 and 10 are not included in the problem set.)

[2 marks each, total 16]

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|---|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| T | 1. $\forall x (Cube(x) \rightarrow \exists y \exists z (Dodec(y) \wedge Dodec(z) \wedge Between(x, y, z)))$ |
| F | 2. $\exists y \exists z (Dodec(y) \wedge Dodec(z) \wedge \forall x (Cube(x) \rightarrow Between(x, y, z)))$ |
| F | 3. $\forall x \forall y ((Cube(x) \wedge Dodec(y) \wedge RightOf(x, y)) \rightarrow Smaller(x, y))$ |
| T | 4. $\exists y (Dodec(y) \wedge \forall x ((Cube(x) \wedge RightOf(x, y)) \rightarrow Smaller(x, y)))$ |
| T | 5. $Cube(a) \wedge \neg \forall x (Dodec(x) \rightarrow Larger(a, x))$ |
| F | 6. $Cube(a) \wedge \forall x (Dodec(x) \rightarrow \neg Larger(a, x))$ |
| F | 7. $\neg \exists x (Cube(x) \wedge \exists y (Dodec(y) \wedge LeftOf(x, y)))$ |
| T | 8. $\exists y (Dodec(y) \wedge \forall x (Cube(x) \rightarrow \neg LeftOf(x, y)))$ |
| F | 9. $\exists v \exists w (Dodec(v) \wedge Dodec(w) \wedge \exists x \exists y (Cube(x) \wedge Cube(y) \wedge x \neq y \wedge Between(x, v, w) \wedge Between(y, v, w)))$ |
| T | 10. $\exists x \exists y (Cube(x) \wedge Cube(y) \wedge x \neq y \wedge \exists v \exists w (Dodec(v) \wedge Dodec(w) \wedge Between(x, v, w) \wedge \exists z \exists u (Dodec(z) \wedge Dodec(u) \wedge Between(y, z, u)))$ |



11.30 Only parts 2, 4 and 7 are part of the problem set.

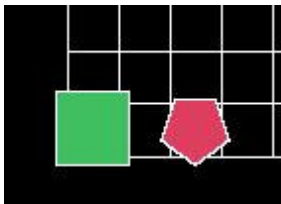
[1 mark each, total 3]

1. $\text{height}(\text{father}(\text{mary})) > \text{height}(\text{mary}) \wedge \neg(\text{height}(\text{father}(\text{mary})) > \text{height}(\text{father}(\text{claire})))$
2. $\exists x (\text{height}(x) > \text{height}(\text{father}(\text{claire})))$
3. $\exists x \text{height}(\text{mother}(x)) > \text{height}(\text{father}(x))$
4. $\forall x \exists y (x \neq y \wedge \text{height}(x) > \text{height}(y))$
5. $\forall x \neg \text{height}(x) > \text{height}(x)$
6. $\forall x ((x \neq \text{jr} \wedge \text{height}(x) > \text{height}(\text{claire})) \rightarrow \text{height}(x) > \text{height}(\text{jr}))$
7. $\forall x (\text{height}(\text{claire}) > \text{height}(x) \rightarrow \exists y (\text{height}(\text{father}(\text{melanie})) > \text{height}(y) \wedge \text{height}(y) > \text{height}(x)))$
8. $\exists x (\text{height}(x) > \text{height}(\text{mother}(\text{father}(\text{jon}))) \wedge \text{height}(\text{father}(\text{mother}(\text{jon}))) > \text{height}(x))$

12.5 [5 marks for world]

The argument is invalid, so here's a counter-example world.

T	1. $\forall y (\text{Cube}(y) \vee \text{Dodec}(y))$
T	2. $\forall x (\text{Cube}(x) \rightarrow \text{Large}(x))$
T	3. $\exists x \neg \text{Large}(x)$
F	4. $\exists x (\text{Dodec}(x) \wedge \text{Small}(x))$



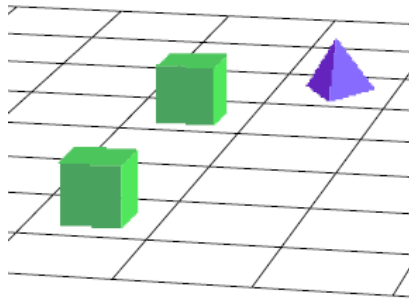
12.7 [5 marks]

The argument is valid, so here's an informal proof.

From P1 we know that everything is either a cube or a dodec. So an arbitrary object, say b , must be either a cube or a dodec. (Note that the world cannot be empty.) But if b is a cube, then it must also be large, from P2. But according to P3, nothing is large. So b can't be a cube; it must be a dodec. But in that case there must be a dodec in the world, which is the conclusion. ■

12.19 [5 marks for world]

The argument is invalid, so here's a counter-example world.



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|---|----|----------------------------------------------------------------------------------------------|
| T | 1. | $\forall x (\text{Cube}(x) \rightarrow \exists y \text{ LeftOf}(x, y))$ |
| T | 2. | $\neg \exists x \exists z (\text{Cube}(x) \wedge \text{Cube}(z) \wedge \text{LeftOf}(x, z))$ |
| T | 3. | $\exists x \exists y (\text{Cube}(x) \wedge \text{Cube}(y) \wedge x \neq y)$ |
| F | 4. | $\exists x \exists y \exists z (\text{BackOf}(y, z) \wedge \text{LeftOf}(x, z))$ |